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Abstract	Algorithms for the synthesis of controllers in the construction of discrete control systems for technological objects are given. Algorithms for constructing state-based stabilizing vector controllers for discrete dynamic objects based on linear matrix inequalities have been developed. The algorithms make it possible to ensure that the requirements for the quality of regulation are met, with the steady values of the controlled parameters exactly falling within the specified tolerances and under the action of limited external disturbances on the system. Algorithms for the synthesis of discrete controllers in nonlinear control systems are proposed, taking into account the delay. The algorithms ensure the asymptotic stability of a closed discrete-continuous system and make it possible to predict the state of the system at each discretization step. Algorithms for estimating the parameters of controller settings based on active adaptation have been developed. Based on the algorithms for the synthesis of the developed controllers, a system for adaptive control of the parameters of the technological process of drying potassium chloride is proposed.
Keywords (separated by '-')	Discrete object - Synthesis algorithms - Stabilizing controller - Linear matrix inequalities - Nonlinear dynamic objects

Controllers Synthesis Algorithms in the Construction of Discrete Control Systems for Technological Objects

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1 Introduction

At present, the automation of technological processes in various industries requires the development of efficient control systems with high speed and accuracy [1–3]. On the other hand, it is necessary to have a priori information about the parameters of the object and disturbing influences for the synthesis of controllers when building high-quality discrete control systems. As a rule, control systems do not have such information. In such cases, it should build the structure of the model. This process depends on the amount of available a priori information about certain parameters of the object. State parameter space methods are often used to solve the problems of synthesizing controllers of discrete control systems for technological objects. Based on the analysis of the theory and the state of construction of discrete control systems for technological objects, it can be concluded that the issues of effective determination of the adjustable parameters of

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adaptive controllers and controller synthesis under conditions when the desired solution is unknown are little studied in this area. Based on the foregoing, when constructing discrete control systems, it is very important to develop methods and algorithms for the synthesis of controllers that allow obtaining stabilizing and effective laws for controlling the state and output, and their practical application [4–18].

2 Objects and Methods

2.1 Algorithms for the Synthesis of Discrete Controllers in Dynamic Control Systems

Let us consider the issues of structural-parametric synthesis of one-dimensional stabilizing controllers of stationary technological control objects. Let us discuss the problem of nonparametric synthesis for constructing discrete action gradient controllers for automatic stabilization of a linear technological control object described in the state space by a difference equation with a retarded argument [5–12].

When digital computer technology is included in the automatic control loop of a computer, the problem of choosing discrete control laws for continuous dynamic objects becomes topical.

Let us turn to a linear stationary discrete object, the behavior of which is described in the state space by a finite difference equation of the type:

$$x_{k+1} = Ax_k + Bu_k, \tag{1}$$

where $u_t \in \Re^m$ is the control action, $x_k \in \Re^n$ is the state of the technological control object, $A \in \Re^{n\times n}$ and $\in \ \Re^{n\times m}$ are the given matrices.

In the case when the variables of the state vector of the dynamic object x_k are measurable, for the controlled object (1) the task of constructing a stabilizing controller according to the state can take place. The latter consists in choosing the following control law in the class of linear feedbacks of the form:

$$u_k = \Theta x_k. \tag{2}$$

Here: $\Theta \in \Re^{\,m+n}$ is the matrix of parameters for the settings of the stabilizing controller, when the state of parameter $x = 0$, „ the closed system (1) and (2), being asymptotically stable in the sense of Lyapunov, will be written as:

$$x_{k+1} = A_c x_k,\ A_c = A + B\,\Theta. \tag{3}$$

One of the potentially possible methods for solving the problem of stabilizing an unstable dynamic object by state is to use linear matrix expressions. The stabilizability of a discrete technological object is identically ensured by the presence of such a quadratic function V_k, which, along each trajectory of motion of a closed system, is ensured by the fulfillment of inequality $\Delta V_k < 0.$. Accordingly, the condition of stabilizability of a linear technological control object (1) is equivalent to the solvability of the Lyapunov inequality:

$$A_c^T X A_c - X < 0 \tag{4}$$

in relation to the unknown matrix $X = X^T > 0$ together with the matrix of parameters of the controller settings Θ:

$$(A + B\,\Theta)^T X\,(A + B\,\Theta) - X < 0 \tag{5}$$

The above inequality is non-linear with respect to the unknown matrices $X = X^T > 0$ and Θ. Therefore, it should be represented in the class of linear matrix inequalities.

Let us turn to possible approaches to solving the presented problem.

The first approach boils down to the fact that the discrete dynamic control object (1) is stabilized only if there is a $(n_x \times n_x)$-matrix $Y = Y^T > 0$ that completely satisfies the linear inequality of the form:

$$W_{B^T}^T\,(AYA^T - Y)\,W_{B^T} < 0, \tag{6}$$

when the columns of matrix W_{B^T} are the basis of the kernel of matrix B^T. In the same case, if inequality (6) is solvable with respect to matrix Y, then the state feedback variables 3 are solutions of the matrix inequality of a linear form:

$$\begin{pmatrix} -Y & A + B\Theta \\ (A + B\Theta)^T & -Y^{-1} \end{pmatrix} < 0 \tag{7}$$

In accordance with a discrete version of the well-known Lyapunov theorem, the stabilizability of the dynamic object under study (1) is equivalent to the existence of a quadratic Lyapunov function $V_k(x_k) = x_k^T X x_k$, when $X = X^T > 0$ is such that the following conditions are satisfied for any trajectory of the considered closed system:

$$\Delta V_k = V_{k+1} - V_k = x_{k+1}^T X x_{k+1} - x_k^T X x_k = x_k^T\,(A_c^T X A_c - X)\,x_k < 0$$

In the case when the linear matrix inequality (6) is solvable, one of the possible solutions can be found in the form of a matrix Y. Then, by substituting Y in (7), a linear matrix inequality is obtained with respect to matrix Θ, solving which, we find the linear feedback parameters systems by state.

The second possible approach is as follows. The resulting inequality (4) is multiplied on the left and right by matrix $X^{-1} > 0$. Then, following the well-known Schur lemma, the last inequality is reduced to the form:

$$X^{-1} A_c^T X A_c X^{-1} - X^{-1} < 0, \quad \begin{pmatrix} -X^{-1} & A_c X^{-1} \\ X^{-1} A_c^T & -X^{-1} \end{pmatrix} < 0, \tag{8}$$

here: $X = X^T > 0$.

Taking into account the form of matrix A_c (3) and denoting $X^{-1} = Y$, from expression (8) one obtains:

$$\begin{pmatrix} -Y & AY + B\Theta Y \\ (AY + B\Theta Y)^T & -Y \end{pmatrix} < 0 \tag{9}$$

A new variable $Z = \Theta Y$ is introduced. Inequality (9) is fixed as a linear matrix inequality with respect to matrices Y and Z and deals with the following inequality:

$$\begin{pmatrix} -Y & AY + BZ \\ (AY + BZ)^T & -Y \end{pmatrix} < 0 \tag{10}$$

If the last inequality (10) is solvable with respect to matrices Y and Z, then the parameters Θ of the linear state feedback are found from the following expression: $\Theta = Z\,Y^{-1}$.

In this paper, we formulate the necessary and sufficient conditions for the existence of linear-quadratic discrete state controllers.

Thus, both possible approaches to solving the problem of structural-parametric synthesis of linear-quadratic stabilizing state controllers are based on the mathematical apparatus of linear matrix inequalities.

In many problems of automatic control of multiply connected technological objects or mobile dynamic systems, it becomes expedient to use several control channels. We will proceed from the assumption that the mathematical model of displacement of a continuous nonlinear control object is formalized by the following vector differential equation:

$$\begin{aligned} &\dot{x}(t) = A(x)x + B(x)\,u[k]\,; \\ &u[k] = const; \quad kT_0 \le t < (k+1)T_0, \end{aligned} \tag{11}$$

where $x \in \Re^n$ is the state space vector, $x(t) = [x_1, x_2, ..., x_n]^T$; $A(x)$,$B(x)$ – functional matrices, dim $(A(x[k]) = (n \times n)$; dim $(B(x[k]) = (n \times m)$; $u[k] \in \Re^m$, m -vector control.

We represent (11) as a vector difference equation:

$$x[k+1] = F(x[k])x[k] + D(x[k])\,u[k], \tag{12}$$

here: $F(x[k])$, $D(x[k])$ are functional matrices of dimensions $(n \times n)$ and $(n \times m)$.

Let us apply the difference approximation procedure to expression (11). For example, according to the Euler formula, we get an expression that connects Eqs. (11) and (12):

$$F(x[k]) = \mathrm{I}^n x[k] + \mathrm{T_0 A}(x[k])\,,\; D(x[k]) = \mathrm{T_0 B}(x[k])\,, \tag{13}$$

where T_0 is the time sampling step.

The task of synthesis, in this particular case, is to determine the control vector $u[k] \in \Re^m$ - one that delivers the transfer of the representing point of the synthesized system from an arbitrary initial state $x^0[k]$ (in the general case $x^0[k] \in \Omega^0$) - to some final state x^f. In this case, it is required that on the trajectories of movement object, the minimum of the optimizing functional was provided:

$$J = \sum_{k=0}^{\infty} \Big(\psi_1[k] M^2 \psi_1[k] + \Delta\psi_1[k] C^2 \Delta\psi_1[k] \Big), \tag{14}$$

here: $M = \left\| m_{ij} \right\|$, $C = \left\| c_{ij} \right\|$ – numerical matrices of dimension $(m \times m)$, $\psi_1[k] \in \Re^m$ – vector aggregated macro variables $\psi_1[k] = \left[\psi_{1,1}, \psi_{1,2} ..., \psi_{1,m} \right]^T$.

In this case, the asymptotic stability of the object's motion in the phase space or in some region of it must be guaranteed. The equations of extremals that would bring the minimum to the functional (14) look as follows:

$$\psi_1[k+1] + \Lambda_1 \psi_1[k] = 0\;, \tag{15}$$

where $\dim(\Lambda_1) = (m \times m)$ and matrix $\Lambda_1 = \left\| \lambda_{1,ij} \right\|$ is such that the solution of difference Eq. (14) is asymptotically stable [17, 18, 20, 21].

The motion of the representative point must satisfy Eq. (14). Since the zero solution of this equation is stable, then the movement at the end of transient processes in system (15) must necessarily satisfy the expression:

$$\psi_1[k] = 0 \tag{16}$$

Thus, the representative point moves from the initial state $x^0[k] \in \Omega^0$ to the intersection of the manifolds (16), $\bigcap\limits_{i=1}^{m} \psi_{1,i}[k] = 0$, and when approaching it, it must move along it to the final state x^f. In this particular case, the problem of vector control synthesis turns into providing the conditions for the projection of the movement of the original object (12) into the subspace of manifolds reflected by expression (14).

Thus, the designed vector controller delivers a solution to the problem of controlling an object by transferring the representative point of a closed system under arbitrary initial and boundary conditions to the origin of the phase space. To top it all, a dynamic discrete controller built on the basis of a series-parallel set of invariant manifolds ensures that the specified technological requirements are met, gives the necessary asymptotic stability to a closed discrete-continuous system, and has the property of predictive prediction of the system behavior at each discretization step.

2.2 Synthesis Algorithms for Optimal Controllers for Discrete Control Systems

Consider a stationary system described by the equations [22–26]:

$$x_{k+1} = Ax_k + Bu_k, \quad z_k = Dx_k, \quad x_k(0) = x_{k|0}, \tag{17}$$

along with functionality:

$$J_l(x_{k|0}) = \min_u \sum_{k=0}^{\infty} \left[\|z_k\|^2 + l^2 \|u_k\|^2 \right], \quad l > 0$$

here u, x and Z are real finite-dimensional control, state, and output vectors.

Consider the nature of the optimal state and control trajectories, say $x_{l|k}$ and $u_{l|k}$, at $l \downarrow 0$.

The bounding paths, say $x_{k|k}$ and $u_{k|k}$, behave as follows. There is a subspace of the state space called a degenerate hyperplane and, regardless of the fact that the initial condition $x_{k|0}$, $x_k(0+)$ is on this degenerate hyperplane, $x_{k|k}$ is displaced relative to it. So around $k = 0$, $x_{k|k}$ is degenerate. Optimal control $u_{k|k}$ is correspondingly degenerate for $k = 0$.

Let us now consider for what initial conditions $x_{k|0}$, $J_l(x_{k|0})$ tends to zero at $l \downarrow 0$. For the case when the transition matrix:

$$G(z) \triangleq D(z - A)^{-1} B$$

is square and invertible, it is proved that

$$\lim_{l \downarrow 0}(k) J_l(x_{k|0}) = 0 \text{ for all, } x_{k|0}, \tag{18}$$

if and only if $G(s)$ is in minimum phase.

We will consider the matrices A, B and D in (17) as linear transformations in real linear finite-dimensional spaces X, U and Z as follows: $A : X \rightarrow X$, $B : U \rightarrow X$, D: $X \rightarrow$ Z.

The optimal control law is $u_k = F_l x_k$, where $F_l = -\frac{1}{l^2} B^T P_l$, and F_l is the only positive semidefinite solution of the algebraic Riccati equation:

$$A^T P_l + P_l A + D^T D = \frac{1}{l^2} P_l B^T B P_l \tag{19}$$

From expression (17) it follows that P_l does not increase when l decreases and that $P = \lim_{l \downarrow 0} P_l$ exists. The closed loop under optimal control is:

$$x_k = (A + BF_l)x_k, \quad x_k(0) = x_{k|0}$$

The transition matrix is $T_{l|k} \overset{\Delta}{=} \exp[k(A + BF_l)]$ and the optimal state and control trajectories are given as:

$$x_{l|k} T_{l|k} x_{k|0}, \quad u_{l|k} = W_{l|k} x_{k|0}, \tag{20}$$

where $W_{l|k} \overset{\Delta}{=} F_l T_{l|k}$.

We define a sequence $\{_{\mathrm{i}}\}$ of a subspace of space X:

$$\Gamma_0 = B, \quad \Gamma_{i+1} = B + A(\ker D \cap {}_i), \; i \geq 0 \tag{21}$$

The sequence given in (21) is non-decreasing, i.e. $\Gamma_i \subset_\Gamma i + 1$ and reaches limit Γ in a finite number of steps. Subspace Γ is the smallest controllable subspace obtained by feeding an output to an input, i.e. is the smallest subspace that is a controllable subspace of $(A + KD, \; B)$ for some μ.

The above algorithms make it possible to increase the accuracy of the parameters of the considered class of dynamic control objects and effectively solve the problems of parametric synthesis of optimal controllers in control systems for industrial technological processes [27–31].

We will assume that the equations of the controller parameters and the measurement process are described by equations of the form [33, 34]:

$$\theta_k = \theta_{k-1} + w_k, \tag{22}$$

$$z_k = H_k \theta_k + \upsilon_k, \tag{23}$$

where θ_k is an n-dimensional vector of controller parameters; z_k – vector of measurements of size m; H_k – measurement matrix of size $m \times n$; w_k and υ_k are normally distributed disturbances with zero averages.

Covariance matrices Q and R are assumed to be unknown but constant over time. Under these conditions, the tasks of adaptive filtering and prediction are, respectively, to obtain a simultaneous estimate $\hat{\theta}_{k|k}$ of vector θ_k and a forward estimate $\hat{\theta}_{k+1|k}$ of vector θ_{k+1} based on observations $Z^k = \{z_k, z_{k-1}, ...\}$.

The solution to these problems, when the matrices Q and R are exactly known, is given by the Kalman filter in the form:

$$\hat{\theta}_{k+1|k} = \hat{\theta}_{k|k}, \hat{\theta}_{k|k} = \hat{\theta}_{k|k-1} + K(z_k - H\hat{\theta}_{k|k-1}),$$

$$K = MH^T(HMH^T + R)^{-1} = PH^TR^{-1}, \tag{24}$$

$$P = (I - KH)M = (M^{-1} + H^TR^{-1}H)^{-1} = M - MH^T(HMH^T + R)^{-1}HM\ M = P + Q,$$

where, $P = E\Big[(\theta_k - \hat{\theta}_{k|k})(\theta_k - \hat{\theta}_{k|k})^T\Big]$, $M = E\Big[(\theta_{k+1} - \hat{\theta}_{k+1|k})(\theta_{k+1} - \hat{\theta}_{k+1|k})^T\Big]$,

Considering the above matrices Q and R as unknown, consider the filter:

$$y_k = D\xi_k, \tag{25}$$

$$\xi_k = y_{k-1} + B(z_k - Hy_{k-1}), \tag{26}$$

coinciding in structure with the optimal filter (24), where D and B are some matrices. In this case, we require that in the process of filter adaptation, convergences with a probability of one are performed

$$D \to I, \quad B \to K, \tag{27}$$

then with the same probability $y_k \to \hat{\theta}_{k+1|k}$, $\xi_k \to \hat{\theta}_{k|k}$

Evaluating the quality of filtering and prediction by unobservable errors.

$$e_k = \theta_k - \xi_k,\ p_k = \theta_{k+1} - y_k \tag{28}$$

form the observed process:

$$\varepsilon_k = H^{-1}z_k - \xi_{k-1}. \tag{29}$$

Taking into account (22), (23), (29), we represent (30) as

$$\varepsilon_k = e_{k-1} + (w_{k-1} + H^{-1}\upsilon_k) \tag{30}$$

and using expansions

$$\theta_k = \sum_{j=0}^{\infty} w_{k-1-j}, \xi_k = \sum_{j=1}^{\infty}(D - BHD)^j Bz_k - j \tag{31}$$

of a subspace of space which are stationary solutions of Eqs. (20) and (23), (24), we obtain:

$$E\Big[\varepsilon_k\varepsilon_k^T\Big] = E\Big[e_{k-1}e_{k-1}^T\Big] + \Big[Q + H^{-1}R(H^{-1})^T\Big].$$

Taking into account the stationarity of the processes, this leads to a relation of the form:

$$J_\varepsilon = S_p E\Big[\varepsilon_k \varepsilon_k^T\Big] = J_e + \text{const}$$

, where $S_p = \left[Q + H^{-1}R(H^{-1})^T\right] = \text{const}$ is a value that does not depend on the filter parameters. The last relation allows us to take J_ε as an auxiliary quality functional and to carry out filter adaptation [32, 35–38].

The above algorithms were used to automate and control the technological process of drying potassium chloride under conditions when the covariance matrices of object noise and measurement noise are a priori unknown and have shown their effectiveness.

3 Results and Discussion

3.1 Application of the Developed Algorithms for the Synthesis of Discrete Controllers in the Tasks of Automation and Control of the Technological Process of Drying Potassium Chloride

The performed formalization of the potassium chloride drying process as a control object made it possible to identify the following main variables characterizing the process under consideration: control parameters $U = (u_1, u_2)$, where u_1 is the temperature of the heat carrier at the dryer inlet; u_2 is the flow rate of the heat carrier at the inlet to the dryer drum; output parameters: $Y = (y_1, y_2)$, where y_1 is the moisture content of the material to be dried at the outlet of the dryer drum, y_2 is the temperature of the heat carrier at the outlet of the dryer drum; uncontrolled disturbances: $w = (w_1, w_2)$, where w_1 is the flow rate of the material to be dried at the inlet to the dryer drum, w_2 is the initial moisture content of the material to be dried at the inlet to the dryer drum. To obtain and practical use of the mathematical model, an industrial experiment was carried out under the conditions of normal operation of the technological process of drying potassium chloride at Dekhkanabad Potash Plant JSC. Based on a preliminary analysis of the dynamic characteristics of the drying process, the registration time T and discretization of implementations Δt of the observed random processes were chosen respectively: $T = 4$ h, $\Delta t = 2$ minutes. A total of 120 measurements were made.

Small deviations of the model from the object and the resulting small delays can lead to loss of stability. The processes occurring in the considered control object can be described by a vector equation of the form:

$$\begin{gathered} x_{k+1} = A_k x_k + F_k x_{k-\tau} + B_k u_k + w_k, \quad y_k = H_k x_k + v_k\,, \\ x_s = \phi_s, \quad s \in [k_0 - \tau,\ k_0]\,, \end{gathered} \tag{32}$$

where B_i – real matrix $n \times m$; u_k is the m-dimensional control vector; τ - constant delays; ϕ_s – initial vector functions; w_k, v_k - vectors of disturbing influences.

Let's connect the regulator to the object (32),

$$u_k = \mathrm{K_k^T y_k} \tag{33}$$

, where K is a $n \times 2l$-dimensional matrix of controller parameters.

Let us assume that the matrices *A, F, B, H* depend on the vector of unknown parameters $\xi \in M$, where *M* is the set that determines the adaptability class of the synthesized system. We will accept the algorithm for setting the controller parameters in the form:

$$\mathrm{k_{k+1}} = \mathrm{k_k} + \Theta_{\mathrm{k}}(\mathrm{y_k}, \tau) \tag{34}$$

where k_i is the i-th column of matrix K; $\Theta_k(y_k, \tau)$, $(k = l,\ldots,m)$ are vector functions, to be determined provided that $\sum\limits_{k=0}^{\infty} \|w_k\|^2 < \infty$, that is, the system (32)-(34) is adaptive in a given class *M*.

To solve the problem under consideration, we use the functional Lyapunov-Krasovsky:

$$V(x_s, \mathrm{k_s}) = x_k^T L_0 x_k + \sum_{k=1}^{m} (\mathrm{k_k} - \mathrm{k_{0k}})^T L_k (\mathrm{k_k} - \mathrm{k_{0k}}) - \gamma \left[\frac{1}{2}\left(x_{k-\tau}^T x_{k-\tau} + x_k^T x_k \right) + \sum_{l=1-\tau}^{-1} x_{k+l}^T x_{k+l} \right],$$

where $\mathrm{k_{0k}}$ is the k-th column of some matrix $\mathrm{K_0}$; L_0, L_k are real symmetric positive definite matrices; $\gamma > 0$.

System (32) will be adaptive in a given class *M* if $\sum\limits_{k=0}^{\infty} \|v_k\|^2 < \infty$, and the algorithm for setting the controller parameters has the form:

$$\mathrm{k_{k+1}} = \mathrm{k_k} - \mathrm{d_k^T z_k\, P_k}\, \psi_{\mathrm{k}}, \quad \mathrm{k} = 1, 2, ..., \mathrm{m} \tag{35}$$

, where *D* is the matrix of column d_k of which is determined by the conditions of the frequency theorem of system stability; P_k – arbitrary positive definite matrices; a

$$\psi^T = y_k^T y_{k-\tau}^T.$$

In this case, it is necessary that the quasipolynomial $\beta(p)\cdot \det\ \mathrm{D^T W(p)}$ has the degree $(n-1)$ and all the roots lay in the left half-plane, and there must also be a diagonal matrix *R* - such that the matrix $\Gamma = \lim\limits_{p\to\infty} pR\,\mathrm{D^T W(p)}$ would be symmetric and positive definite, where $W(p) = L\left(\lambda I_n - A - Fe^{-p\tau}\right)^{-1} b = \frac{\alpha(p)}{\beta(p)}$, $W(p)$ is the transfer matrix of the object; $\beta(p) = \det\left(pI - A - Fe^{-p\tau}\right)$ - characteristic quasi-polynomial of the object; $\alpha(p)$ is an l-dimensional vector whose components are some quasi-polynomials.

The assessment of the quality of the transient process in the system will be determined by the expression:

$$\sum_{k=0}^{\infty} x_k^T Q x_k \le V(\mathrm{k_0}, \phi_{\mathrm{s}})$$

To solve the problem of estimating the state vector of the object under consideration, we use the methods of the theory of dynamic estimation and filtering:

$$\hat{x}_{k+1} = A_k\hat{x}_k + F_k\hat{x}_{k-\tau} + B_k u_k + K_k^1[y_k - \mathrm{K_k \hat{x}_k} - \mathrm{K_k B_k u_k}] -$$
$$-\frac{\tau}{n}\left[\frac{1}{2}\left(K^2_{k,-\tau}[y_{k-\tau} - \mathrm{K_{k-\tau}\hat{x}_{k-\tau}}] + \mathrm{K^2_{k,0}}[\mathrm{y_i} - \mathrm{K_k\hat{x}_k}]\right) + \sum_{l=1-\tau}^{-1} K^2_{k,l}[y_{k+l} - \mathrm{K_{k+l}\hat{x}_{k+l}}]\right]. \tag{36}$$

The first term in (36) represents the a priori estimate of the object's state vector, the second - the same estimate, but with a delay, the third term - the correction to the estimate of the object's state vector, equal to the weighted difference between the a priori estimate $\mathrm{K_k\hat{x}_k}$ of the object's output signal and the measured value y_k of this signal, the fourth term is the distributed correction to the estimate of the state vector with delay [39–41].

The gains K_k^1 and K_k^2 are given by the equations

$$K_k^1 = [P_k - P_{k,0}]\,\mathrm{K_k^T R_k^{-1}}, \quad \mathrm{K^2_{k,s}} = \mathrm{P_{k,s} K^T_{k+s} R^{-1}_{k+s}}\,. \tag{37}$$

The equations for the covariance matrices of estimation errors will take the form:

$$P_{k+1} = A_k P_k + P_k A_k^T - P_k \mathrm{N_k P_k} + \mathrm{F_k} + \mathrm{P_{k,0} N_k P_{k,0}} -$$
$$-\frac{\tau}{n}\left[\frac{1}{2}\left(P_{k,-\tau}\mathrm{N_{k,-\tau}P_{k,-\tau}} + \mathrm{P_{k,0}N_{k,0}P_{k,0}}\right) + \sum_{l=1-\tau}^{-1} P_{k,l}\mathrm{N_{k,l}P_{k,l}}\right] + A_k^1 P_{k,-\tau} + P_{k,-\tau}A_k^{1\,T},\ P_0 = P_{k_0}\,,$$
$$\frac{\partial P_{k,s}}{\partial t} = \frac{\partial P_{k,s}}{\partial s} + [A_k - [P_k - P_{k,0}]\,\mathrm{N_k}]\mathrm{P_{k,s}} + \mathrm{A_k^1 P_{k,-\tau,s}} +$$
$$+\frac{\tau}{n}\left[\frac{1}{2}\left(P_{k,-\tau}\mathrm{N_{k-\tau}P_{k,-\tau,s}} + \mathrm{P_{k,0}N_k P_{k,0,s}}\right) + \sum_{l=1-\tau}^{-1} P_{k,l}\mathrm{N_{k+l}P_{k,l,s}}\right], \quad P_{k_0} \equiv 0, \quad s \in [-\tau, 0)\,,$$
$$\frac{\partial P_{k,r,s}}{\partial t} = \frac{\partial P_{k,r,s}}{\partial r} + \frac{\partial P_{k,r,s}}{\partial s}, \quad P_{k_0,r,s} \equiv 0,\ r,s \in [-\tau, 0), \quad P_{k,r,s} = P^T_{k,s,r},\ \mathrm{N_k} = \mathrm{H_k^T R_k^{-1} H_k} \tag{38}$$

Based on relations (31)-(34) and the developed algorithms for the synthesis of an adaptive control system, we can propose the following version of an adaptive control system for the drying of potassium chloride (Fig. 1), which consists of a controlled process 1, a controller 2, an optimal estimation unit and identification 3, optimal control formation unit 4, controller tuning and adaptation unit 5.

The adaptability feature allows you to use the same set of algorithms to control different processes. It should be said that the adaptive evaluation block can be expressed in the form of software or separate small blocks in a computer. Based on the foregoing, a structural-functional scheme for controlling the dryer drum is proposed in the following form (Fig. 2).

The proposed structural-functional scheme for controlling a programmable logic controller (PLC) receives signals from controlled sensors in the dryer drum: temperature sensors TT01, TT02 and TT03, flow sensor FT01 and pressure sensor PT01. They, in turn, are stored in the database archive. One of the distinguishing features of PLC sensors, in addition to accepting signals, also includes some control signals in various operating modes, namely: manual or automatic control mode selection, system start or stop, and emergency stop.

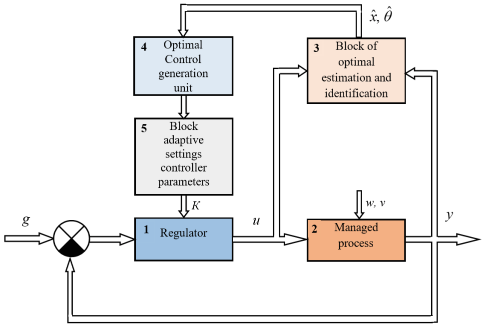

Fig. 1. The structure of the adaptive control system for the drying of potassium chloride: g – setting influences; w, v are disturbances; u – control actions; $\widehat{\theta}$ – estimates of object parameters; $\widehat{x}$ – estimates of the current state of the object; K – adjustable parameters of the controller; y – output signal of the object.

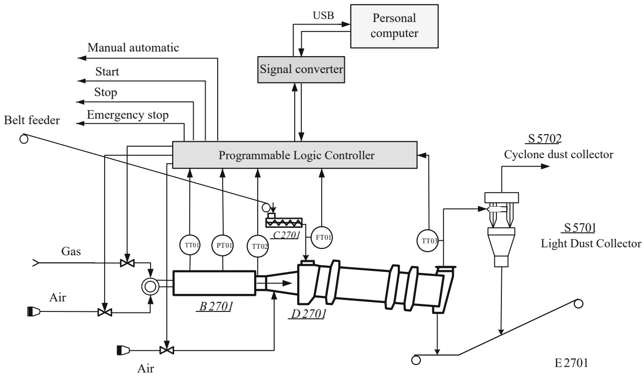

Fig. 2. Structural and functional diagram of the dryer drum control: D2701 – drum dryer, V2701 – air heating system, E2701 – drum cooler, S5702 – cyclone dust collector, S5702 – light dust collector, C2701 – batcher, TT01, TT02, TT03 – temperature sensors, PT01 – pressure sensor, FT01 – flow sensor.

Software has been developed that implements the algorithm of the circuit shown in Fig. 2. On the basis of the developed software, a numerical simulation of the control process was carried out based on the algorithm (38) taking into account (33). So, for example, the following figures show the implementation of the control action and the output variable of the process under consideration through the channel $u_1 \rightarrow y_1$ and $u_2 \rightarrow y_2$ (Fig. 3).

Thus, the use of the proposed adaptive control system for the process of drying potassium chloride makes it possible to stabilize the technological regimes of the process and increase the productivity of the drying plant by an average of 1.8%, as well as save gas consumption.

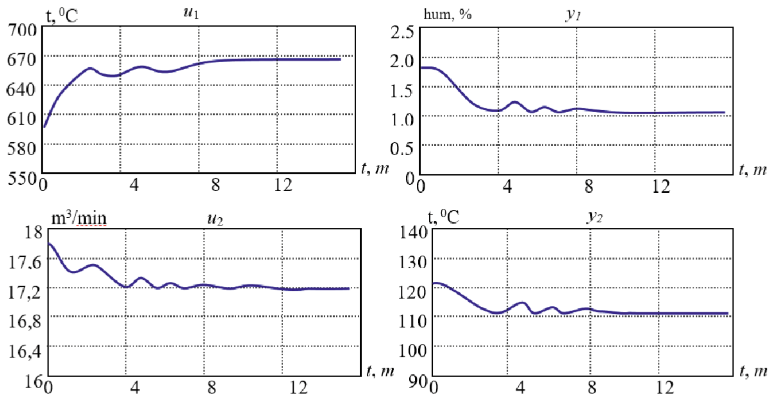

Fig. 3. Implementation of the control action and the output variable of the process under consideration through the channels $u_1 \to y_1$ and $u_2 \to y_2$.

4 Conclusion

As a result, the following scientific results were obtained: on the basis of linear matrix inequalities, the parameters of the controller were determined, which allow stabilizing discrete objects in terms of state and ensuring acceptable accuracy of the control system regulation processes in the presence of unmeasured external disturbances; algorithms for the synthesis of discrete controllers in nonlinear control systems are proposed, taking into account the delay. The algorithms ensure the asymptotic stability of a closed discrete-continuous system and make it possible to predict the state of the system at each discretization step; algorithms for estimating the parameters of controller settings based on active adaptation have been developed. Based on the algorithms for the synthesis of the developed controllers, a system for adaptive control of the parameters of the technological process of drying potassium chloride is proposed. The proposed adaptive control system makes it possible to stabilize the technological regimes of the process and increase the productivity of the plant by an average of 1.8%, as well as save gas consumption.

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Chapter 36

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